

The hydrogen atom

From the Hamiltonian to bound states

Time evolution

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi,$$
$$\hat{H} = -\frac{\hbar^2}{2m} \nabla^2 - \frac{e^2}{4\pi\epsilon_0 r}.$$

The spatial equation

$$-\frac{\hbar^2}{2m} \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right) - \frac{e^2 \psi}{4\pi\epsilon_0 \sqrt{x^2 + y^2 + z^2}} = E\psi.$$

Separation of variables

$$\psi_{n\ell m}(r, \theta, \phi, t) = R_{n\ell}(r) Y_{\ell}^m(\theta, \phi) e^{-iE_n t/\hbar}.$$

The energy spectrum

$$E_n = -\frac{me^4}{2(4\pi\epsilon_0)^2 \hbar^2 n^2}, \quad n = 1, 2, 3, \dots$$